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EFFECTS OF VARIABLE VISCOSITY AND THERMAL CONDUCTIVITY ON MHD CONVECTIVE HEAT TRANSFER OF IMMISCIBLE FLUIDS IN A VERTICAL CHANNEL

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ABSTRACT

The effects of variable viscosity and thermal conductivity on the MHD flow of two immiscible fluids in a vertical channel are presented. The differential equations governing the flow, heat transfer of the problem are transformed into dimensionless form of ordinary differential equations by using similarity substitutions. The governing boundary value problems so obtained are then solved numerically using Runge-Kutta $6th$ order method. The effects of pertaining parameters on velocity and temperature field are studied and results are presented graphically. The co-efficient of skin-friction and Nusselt number are also computed and presented in tabular form

KEYWORDS: Variable viscosity, Variable thermal conductivity, MHD, Free convection, Immiscible fluids, Runge-Kutta 6th order method.

I. INTRODUCTION

Heat and mass transfer processes has a wide range of applications in Engineering Sciences. Convection problems of electrically conducting fluid have got much importance due to various applications in Geophysics and Engineering, Missile technology etc. MHD deals with the motion of an electrically conducting fluid in the presence of a magnetic field. There are various number of examples of application of MHD. MHD convection problems are widely applied and studied in agriculture, engineering, petroleum industries, Medicine and Biology. A number of works were done on the generalization of flow and heat transfer solution with regards to Magneto hydrodynamics.

For many years, Scientists and Engineers have been interested in two phase flows which arise in many industrial applications. The two-phase fluid flow phenomena are important in pipe flows, fluidized beds, sedimentation, gas purification, transport processes and shock waves. Srinivas et al.[1]. Studied the effects of radiative heat transfer on entropy generation in flow of two immiscible non-Newtonian fluids between two horizontal parallel plates. Ramana Murthy and Srinivas[2] presented the heat transfer by the first and second laws of thermodynamics for the flow of two immiscible couple stress fluids inside a horizontal channel under the action of an imposed transverse magnetic field. Mehdi-Nejad et al.[3] studied a method to calculate heat transfer across an interface separating immiscible fluids.

In a typical operating situation lubricants can be subjected to extreme conditions, like high temperatures, pressure and shear rate. External heating and high shear rates can cause to high temperatures being generated within the fluid. This may have a major effect on the fluid properties as presented by Myers et al.[4]. Hazarika and Santana Hazarika [5] studied the effects of variable viscosity and thermal conductivity on the MHD mixed convective boundary layer flow and heat transfer over a stretching surface in the presence of heat radiation and concluded that viscosity is proportional to velocity and magnetic intensity decreases as velocity increases.

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Surajit Dutta [6] emphasized the effects of variable viscosity and thermal conductivity and the micro inertia density is assumed to be variable on the MHD flow of micro polar fluid on a continuous moving surface in the presence of a transverse magnetic field have studied considering the viscosity and thermal conductivity as the inverse linear functions of temperature. Subhas Abel[7] Presented MHD boundary layer flow and heat transfer in a viscoelastic fluid over a stretching sheet with variable thermal conductivity in presence of radiation and the non-uniform heat source. Salawu and Dada[8] investigated the effects of variable viscosity and thermal conductivity on radiative heat transfer with inclined magnetic field and dissipation in a Darcy medium . Anjali Devi and Prakash[9] presented the problem of temperature dependent viscosity and thermal conductivity effects on hydromagnetic flow over a slendering stretching sheet have been analyzed. Lai and Kulacki[10] presented the effect of variable viscosity along a vertical plate. Hossain et.al[11] have studied the effect of a temperature dependent viscosity on natural convection flow of viscous incompressible fluid from a vertical wavy surface has been investigated using an implicit finite difference method. S.Mukhopadhyay and G.C.Layek[12] studied free convective boundary layer flow and heat transfer of a fluid with variable viscosity over a porous stretching vertical surface in presence of thermal radiation. Oluwole Daniel Makinde[13] analyzed the combined effects of radiation, temperature dependent viscosity, suction and injection on thermal boundary layer over a permeable flat plate with a convective heat exchange at the surface.

In many branches of science and rapid growth in fluid mechanics research the significant importance has been shown on the study of convection in immiscible fluid flows . The effect of variable fluid viscosity and thermal conductivity on the heat transfer through the immiscible fluid flows has not been studied as per the best of my knowledge and available literature. In view of these, the present problem is concentrated on the fluid properties which depend on high temperature. Thus the main motivation of the present paper is to study the effects of variable fluid viscosity and thermal conductivity on MHD boundary layer flow and heat transfer of fluid in a vertical channel. Consideration of temperature-dependent viscosity, thermal conductivity and magnetic parameter yields a highly non-linear coupled system of partial differential equations. The coupled non-linear partial differential equations governing the problem are reduced to a system of coupled highly non-linear higherorder ordinary differential equations by applying suitable similarity transformations. The system of higher order ordinary differential equations is then solved by employing Runge-Kutta-6th order method.

II. MATHEMATICAL FORMULATION

The walls of the vertical channel are placed at $Y = -h_l$ and $Y = h_2$ along Y-direction initially as shown in Figure 1 and both plates are isothermal with different temperatures T_I and T_2 respectively. The distance $-h_1 \le Y \le 0$ represents region-1 and the distance $0 \le Y \le h_2$ represents region-2 where the first region is filled with the fluid having density ρ_1 , viscosity μ_1 and the second region is filled with another fluid having density ρ_2 viscosity

 μ_2 . The fluid flow in the channel is due to buoyancy forces.

The following assumptions are considered to get the governing equations for the problem :

- 1. The flow is assumed to be one-dimensional, steady, laminar, immiscible and incompressible.
- 2. The transport properties of both fluids are constant.
- 3. The fluid flow is due to buoyancy forces
- 4. The fluid flow is fully developed.
- 5. The flow, temperature and species concentration are assumed to continuous at the interface.
- 6. Each of the walls are isothermal and having constant species concentration and $T_1 > T_2$, $C_1 > C_2$
- 7. The flow is assumed to follow Boussinesq approximation.

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Governing Equations

Region-1:
\n
$$
\frac{dU_1}{dY} = 0
$$
 [Continuity]
\n
$$
\rho_1 = \rho_0 [1 - \beta_{1T} (T_1 - T_0)]
$$
 [State] (2)

$$
\frac{1}{\rho_1} \left\{ \frac{\partial \mu_1}{\partial Y} \frac{\partial U_1}{\partial Y} + \mu_1 \frac{\partial^2 U_1}{\partial Y^2} \right\} + g\beta_{1T} (T_1 - T_0) - \frac{\sigma \beta_0^2 U_1}{\rho} = 0 \quad \text{[Momentum]}
$$
\n(3)

$$
U_1 \frac{\partial T_1}{\partial Y} = \frac{1}{\rho C_{\rho}} \left[\frac{\partial}{\partial y} k \frac{\partial T_1}{\partial Y} + \mu_1 \left(\frac{\partial U_1}{\partial Y} \right)^2 \right]
$$
 [Energy] (4)

Region-2:

$$
\frac{dU_2}{dY} = 0
$$
 [Continuity] (5)

$$
\rho_2 = \rho_0 [1 - \beta_{2T} (T_2 - T_0)] \tag{6}
$$

$$
\frac{1}{\rho_2} \left[\frac{\partial \mu_2}{\partial Y} \frac{\partial U_2}{\partial Y} + \mu_2 \frac{\partial^2 U_2}{\partial Y^2} \right] + g \beta_{2T} (T_2 - T_0) - \frac{\sigma \beta_0^2 U_2}{\rho} = 0 \text{ [Momentum]}
$$
\n(7)

$$
U_2 \frac{\partial T_2}{\partial Y} = \frac{1}{\rho_2 C_{P_2}} \left[\frac{\partial}{\partial Y} K_2 \frac{\partial T_2}{\partial Y} + \mu_2 \left(\frac{\partial U_2}{\partial Y} \right)^2 \right]
$$
 [Energy] (8)

The above system of Eqs. (1) to (8) are solved by considering the following boundary and interface conditions as mentioned by Arimen[14].

$$
U_1 = 0 \text{ at } Y = -h_1 \text{ , } U_2 = 0 \text{ at } Y = h_2 \text{ , } U_1(0) = U_2(0), T = T_1 \text{ at } Y = -h_1 \text{ , } T = T_2 \text{ at } Y = h_2 \text{ , } T_1(0) = T_2(0),
$$

\n
$$
\frac{dU_1(0)}{dY} = 0 \text{ , } \frac{dU_2(0)}{dY} = 0 \text{ , } \frac{dT_1(0)}{dY} = 0 \text{ , } \frac{dT_2(0)}{dY} = 0.
$$

The following variables are used to make the system of Equations. (1) to (8) in to dimensionless form:

$$
y = \frac{Y}{h_1} \text{ (Region 1), } u_1 = \frac{U_1}{U_0}, \ \theta_1 = \frac{T_1 - T_0}{\Delta T}, M = \frac{\sigma B_0^2 h_1^2}{\mu_\infty} \text{ (Magnetic field parameter)}, Gr = \frac{g \beta_{1T} \Delta T h_1^3}{v_1^2}
$$

(Grashof number),
$$
v_1 = \frac{\mu_{\infty}}{\rho_1}
$$
, $R = \frac{U_0 h_1}{v_1}$ (Reynolds number), $Pr = \frac{v_1}{\alpha_0}$, $\alpha_1 = \frac{k_1}{\rho_1 C \rho_1}$, $Ec = \frac{U_0^2}{C \rho_1 \Delta T}$ (Eckert

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number),
$$
y = \frac{Y}{h_2}
$$
 (Region 2), $u_2 = \frac{U_2}{U_0}$, $\theta_2 = \frac{T_2 - T_0}{\Delta T}$, $h = \frac{h_1}{h_2}$, $b = \frac{\beta_{1T}}{\beta_{2T}}$, $\rho = \frac{\rho_1}{\rho_2}$, $CP = \frac{Cp_1}{Cp_2}$, $v_2 = \frac{\mu_{\infty}}{\rho_2}$,
 $\alpha_{\infty} = \frac{k_2}{\rho_2}$.

As the crux of the work is to study the effect of variable viscosity and variable thermal conductivities, it is considered in the function of temperature as
$$
\mu_1 = -\frac{\mu_\infty \theta_{r1}}{\theta_1 - \theta_{r1}}
$$
 and $\mu_2 = -\frac{\mu_\infty \theta_{r2}}{\theta_2 - \theta_{r2}}$ for region 1 and 2 respectively where θ_{r1} and θ_{r2} are variable viscosity parameters of the region 1 and region 2 respectively. The variable thermal conductivities for both the regions are taken as $\alpha_1 = \alpha_0 (1 + \beta_1 \theta_1)$ and $\alpha_2 = \alpha_0 (1 + \beta_2 \theta_2)$ where β_1 and β_2 are variable conductivity parameters of region 1 and region 2 respectively.

Hence the governing equations will become in to non dimensional form as : Region-1:

$$
\frac{\theta_{r1}}{(\theta_1 - \theta_{r1})^2} \frac{\partial \theta_1}{\partial y} \frac{\partial u_1}{\partial y} - \frac{\theta_{r1}}{(\theta_1 - \theta_{r1})} \frac{\partial^2 u_1}{\partial y^2} + \frac{G_r}{R} \theta_1 - M u_1 = 0
$$
\n(9)

$$
\beta_1 \left(\frac{\partial \theta_1}{\partial y}\right)^2 + \left(1 + \beta_1 \theta_1\right) \frac{\partial^2 \theta_1}{\partial y^2} - P_r E_c \left(\frac{\theta_{r1}}{\theta_1 - \theta_{r1}}\right) \left(\frac{\partial u_1}{\partial y}\right)^2 = 0
$$
\n(10)

Region -2

$$
\frac{\theta_{r2}}{(\theta_2 - \theta_{r2})^2} \frac{\partial \theta_2}{\partial y} \frac{\partial u_2}{\partial y} - \frac{\theta_{r2}}{(\theta_2 - \theta_{r2})} \frac{\partial^2 u_2}{\partial y^2} + \frac{1}{\rho b h^2} \frac{G_r}{R} \theta_2 - \frac{M u_2}{h^2} = 0
$$
\n(11)

$$
\beta_2 \left(\frac{\partial \theta_2}{\partial y}\right)^2 + \left(1 + \beta_2 \theta_2\right) \frac{\partial^2 \theta_2}{\partial y^2} - \left(\frac{\theta_{r2}}{\theta_2 - \theta_{r2}}\right) \rho P_r E_c \left(\frac{\partial u_2}{\partial y}\right)^2 = 0 \tag{12}
$$

The non dimensional boundary and interface conditions thus formed are:

$$
u_1(-1) = 0, u_2(1) = 0, u_1(0) = u_2(0), u'_1(0) = 0, \theta'_1(0) = 0
$$

$$
\theta_1(-1) = 1, \theta_2(1) = 0, \theta_1(0) = \theta_2(0)
$$
 (13)

III. SOLUTION OF THE PROBLEM

The governing partial differential Equations (9) – (12) of momentum and energy are reduced into non-linear differential equations using the set of similarity variable (13). The obtained equations constitute a system of highly non-linear coupled boundary value problem and are solved numerically by using Runge Kutta $6th$ order method with the help of software Mathematica 10.4. It is observed that a very good agreement has been achieved with their results. The nusselt number and shearing stress on both walls are calculated using the expressions:

$$
Nu_1 = \left[\frac{\partial \theta_1}{\partial y}\right]_{y=1}, Nu_2 = \left[\frac{\partial \theta_2}{\partial y}\right]_{y=1}, St_1 = \left[\frac{\partial u_1}{\partial y}\right]_{y=-1}, St_2 = \left[\frac{\partial u_2}{\partial y}\right]_{y=1}.
$$

IV. SOLUTION OF THE PROBLEM

The numerical solutions of the system of equations are analyzed for different values of the governing parameters and the results are presented graphically. Grashof number (Gr), Reynolds number (R), Magnetic field parameter (M), Eckert number (Ec), variable viscosity parameters θ , and variable thermal conductivity parameters (β_1, β_2) are fixed as Gr=3, R=3, M=3, Ec=0.035, θ_{r1} =-0.8, β_1 = 0.7, β_2 = 0.5 for all the profiles excepting the varying parameter.

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Fig. 4: Velocity Profiles for different M Fig. 5: Velocity Profiles for different Ec -1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1 -1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1

Fig. 8: Temperature Profiles for different θr Fig. 9: Temperature Profiles for different β

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The velocity profiles are illustrated in Figures 2 to 7. The Grashof number for heat transfer is defined to be the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As Gr increases the velocity increases in both regions because of the enhancement of thermal buoyancy force as observed in figure 2. The influence of Reynolds number on velocity is shown in figure 3 . As Reynolds number is the ratio of inertial forces to viscous forces within a fluid which is subjected to relative internal movement due to different fluid velocities, the increase of this number leads to decay in the velocity. Figure 4 represents the effect of Magnetic field parameter; it shows that the momentum is decreasing due to the Lorentz forces which have the capacity to act against the flow. The effect of the Eckert number on velocity is shown in figure 5, Ec is proportional to velocity and not shown significant effect. The effect of variable viscosity parameters is studied by taking different combinations of θ_r for both the regions as θ_r increase the velocity in both the regions increases which is shown in figure 6. The effect of thermal conductivity parameters β_1, β_2 on velocity is depicted in the figure 7, it shows for different combinations of β the velocity is reduced in both the boundaries and found to be more in second region. Figures 8 to 13 depict the effect on temperature for all the pertinent parameters.

Table. 1: Nusselt number and Shear stress

Table 1 shows the Shear stress values and nusselt numbers with the effects of all governing parameters. From this table, it is observed that the absolute Shear stress increases with increase of Gr on both the boundaries $y = -1$ and $y = 1$ because of buoyancy forces and similar nature is identified with Ec. The reverse effect is identified with R and M. For variations of θ_r the shear stress is increasing on the left and decrease on the right boundary, whereas for β_1, β_2 the effect is reversal. The Nusselt number i.e. rate of heat transfer increases on the both the boundaries for the parameters Gr and Ec. For the variations of the Reynolds Number and magnetic field parameter the rate of heat transfer decrease on both the boundaries.

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V. CONCLUSIONS

- 1. The variation of viscosity and thermal conductivity enhances the flow significantly.
- 2. The thermal conductivity variation is could not enhance the temperature notably.
- 3. The shear rate is more on the hot plate than the cold plate. The absolute rate of heat transfer is more on cold plate than on hot plate.
- 4. The variable viscosity and thermal conductivity significantly enhances the rate of shear stress and rate of heat transfer.
- 5. The increase in thermal conductivity and viscosity of the fluid transfer the heat more significantly even in immiscible flows.

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